Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 3 3 5 3 6 0 8 2 6 2 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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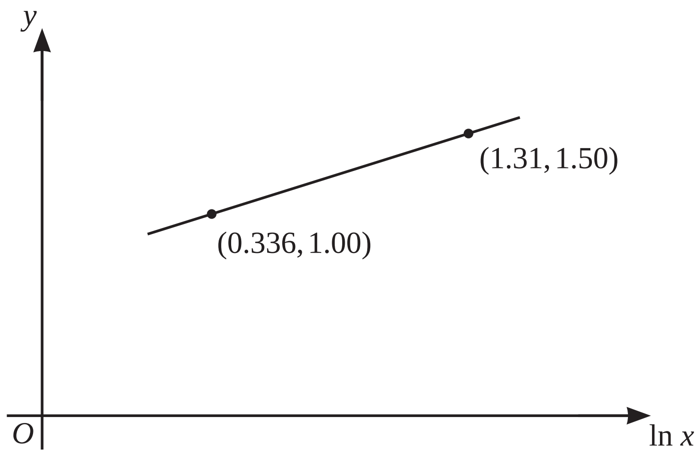
1 Expand $(3+x)(1-2x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Solve the equation $\ln(x-5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

3



The variables x and y satisfy the equation $a^y = bx$, where a and b are constants. The graph of y against $\ln x$ is a straight line passing through the points $(0.336, 1.00)$ and $(1.31, 1.50)$, as shown in the diagram.

Find the values of a and b. Give each value correct to the nearest integer. [4]

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4 The complex number u is given by $u = -1 - \mathrm{i}\sqrt{3}$.

(a) Express u in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ. [2]

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The complex number v is given by $v = 5\left(\cos\frac{1}{6}\pi + \mathrm{i}\sin\frac{1}{6}\pi\right)$.

(b) Express the complex number $\frac{v}{u}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. [2]

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5 The equation of a curve is $y = \dfrac{e^{\sin x}}{\cos^2 x}$ for $0 \leqslant x \leqslant 2\pi$.

Find $\dfrac{dy}{dx}$ and hence find the x-coordinates of the stationary points of the curve. [7]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec}\frac{1}{2}x = e^x - 3$ has exactly one root, denoted by α, in the interval $0 < x < \pi$. [2]

(b) Verify by calculation that α lies between 1 and 2. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula

$$x_{n+1} = \ln(\operatorname{cosec}\tfrac{1}{2}x_n + 3)$$

converges, then it converges to α. [1]

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(d) Use this iterative formula with an initial value of 1.4 to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(e) State the minimum number of calculated iterations needed with this initial value to determine α correct to 2 decimal places. [1]

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7 **(a)** On a single Argand diagram sketch the loci given by the equations $|z-3+2i|=2$ and $|w-3+2i|=|w+3-4i|$ where z and w are complex numbers. [4]

(b) Hence find the least value of $|z-w|$ for points on these loci. Give your answer in an exact form. [2]

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8 Use the substitution $u = 1 - \sin x$ to find the exact value of

$$\int_{\pi}^{\frac{3}{2}\pi} \frac{\sin 2x}{\sqrt{1-\sin x}}\,dx.$$

Give your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

9 The equations of two straight lines l_1 and l_2 are

$$l_1: \quad \mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad l_2: \quad \mathbf{r} = -\mathbf{i} - \mathbf{j} - \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}),$$

where a is a constant.

The lines l_1 and l_2 are perpendicular.

(a) Show that $a = 4$. [1]

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The lines l_1 and l_2 also intersect.

(b) Find the position vector of the point of intersection. [4]

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The point A has position vector $-5\mathbf{i}+\mathbf{j}-9\mathbf{k}$.

(c) Show that A lies on l_1. [2]

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The point B is the image of A after a reflection in the line l_2.

(d) Find the position vector of B. [2]

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10 **(a)** Given that $2x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{2}{1+4x^2}$. [3]

(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x\tan^{-1}(2x)\,dx$. [7]

11 In a field there are 300 plants of a certain species, all of which can be infected by a particular disease. At time t after the first plant is infected there are x infected plants. The rate of change of x is proportional to the product of the number of plants infected and the number of plants that are **not** yet infected. The variables x and t are treated as continuous, and it is given that $\frac{dx}{dt} = 0.2$ and $x = 1$ when $t = 0$.

(a) Show that x and t satisfy the differential equation

$$1495\frac{dx}{dt} = x(300 - x).$$ [2]

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(b) Using partial fractions, solve the differential equation and obtain an expression for t in terms of a single logarithm involving x. [9]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.